

# Methodologies for the design of LCC voltage-output resonant converters

M.P. Foster, H.I. Sewell, C.M. Bingham, D.A. Stone and D. Howe

**Abstract:** The paper presents five structured design methodologies for third-order LCC voltage-output resonant converters. The underlying principle of each technique is based on an adaptation of a FMA equivalent circuit that accommodates the nonlinear behaviour of the converter. In contrast to previously published methods, the proposed methodologies explicitly incorporate the effects of the transformer magnetising inductance. Furthermore, a number of the methodologies allow the resonant-tank components to be specified at the design phase, thereby facilitating the use of standard off-the-shelf components. A procedure for sizing the filter capacitor is derived, and the use of error mapping, to identify parameter boundaries and provide the designer with a qualitative feel for the accuracy of a proposed converter design, is explored.

## List of symbols

$C_f$	filter capacitance (F)
$C_p$	parallel capacitance (F)
$C_s$	series capacitance (F)
$C_{tot}$	total equivalent-circuit capacitance (F)
$C_\eta$	equivalent load capacitance (F)
$P_f$	power factor
$\hat{I}_{in}$	peak input current (A)
$I_{out}$	output current (A)
$k$	rectifier coefficient
$L_m$	transformer magnetising inductance (H)
$L_s$	series inductance (H)
$M_i$	resonant circuit to output-current ratio
$M_v$	voltage-conversion ratio
$n$	transformer-turns ratio
$P_{out}$	output power (W)
$Q$	equivalent-circuit quality factor
$Q_{Cf}$	charge flowing through $C_f$ (C)
$R_L$	load resistance ( $\Omega$ )
$R_\eta$	equivalent load resistance ( $\Omega$ )
$V_b$	rectifier input voltage (V)
$v_{cp}$	parallel capacitor voltage (V)
$V_{dc}$	DC input voltage (V)
$V_{out}$	output voltage (V)
$Z_{tot}$	total equivalent-circuit impedance ( $\Omega$ )
$\alpha$	percentage-ripple-voltage specification
$\delta_1$	rectifier nonconduction period (s)
$\delta_2$	$C_f$ charging duration (s)
$\theta_1$	rectifier nonconduction angle ( $^\circ$ )
$\omega_0, f_0$	resonant frequency ( $\text{rads}^{-1}$ , Hz)

$\omega_n, f_n$	normalised switching frequency ( $\text{rads}^{-1}$ , Hz)
$\omega_s, f_s$	switching frequency ( $\text{rads}^{-1}$ , Hz)

## 1 Introduction

Increasingly, resonant power converters are becoming the preferred technology for a wide variety of industrial, commercial- and consumer-product applications, owing mainly to their high power density. In part, this is facilitated by a reduction in component size, due to the use of high switching frequencies and the reduced heat-sinking requirements, which results from their increased efficiency. Additionally, resonant converters offer the possibility of exploiting transformer parasitics, such as the magnetising inductance, leakage inductance and interwinding capacitance, directly in the resonant tank circuit, thereby reducing the need for discrete components. Indeed, several component manufacturers have recognised the commercial potential of resonant converters, and are producing dedicated resonant-converter-controller integrated circuits.

Although many resonant-converter topologies have been investigated in the literature, it is the LCLC family of converters that has received the most attention. Particular members of the LCLC family offer significantly superior open- and short-circuit behaviour compared with second-order counterparts, with some also exhibiting load-independent operating points [1, 2]. However, in contrast to second-order variants, the analysis and design of these third-order converters is complicated by the requirement to model more than one useful mode of operation.

Although the use of fundamental-mode-approximation (FMA) techniques for the analysis and design of resonant converters [1–3] has been widespread, they suffer from a number of limitations. First, as the switching frequency moves away from the resonant frequency, the tank-current waveform becomes increasingly triangular, thereby invalidating the sinusoidal-waveform assumption which is implicit in the use of FMA. Secondly, the number of circuit states that exist during a single switching cycle can change with the operating condition. The LCC current-output

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IEE Proceedings online no. 20050357

doi:10.1049/ip-epa:20050357

Paper first received 6th September and in final revised form 6th December 2005

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converter, for example, exhibits significant nonlinear behaviour when heavily loaded [4], leaving the designer with a degree of uncertainty as regards the appropriateness of the chosen equivalent circuit. In an attempt to address such issues, ‘modified’ FMA equivalent circuits, that more accurately represent the waveshape of a particular variable, have been developed. In [4], for instance, an equivalent-circuit model for the LCC current-output converter operating under heavy- and intermediate-load conditions was developed, by extracting the fundamental component from an equation which described the nonsinusoidal parallel capacitor voltage. Other design/analysis methodologies based on state-space models have also been reported, although they generally require the solution of nonlinear simultaneous equations, which often requires recourse to numerical methods and/or design charts [5, 6].

In this paper, five novel design-synthesis methodologies for LCC voltage-output converters are presented. All the techniques enable the values of the resonant-circuit components to be determined given a specified voltage-conversion factor, and a further specification particular to the chosen methodology. The equivalent circuit is defined entirely in closed form, thereby making the procedures deterministic, and recourse to interpolation or complex numerical solutions is not required. Further, a procedure for sizing the filter capacitor is derived based on the equivalent-circuit model. The paper also introduces the concept of confidence mapping to provide a mechanism for assessing the accuracy of the results generated from a particular design procedure, and give the designer a means of fine tuning a converter’s performance. Thus, it has the potential to reduce the design effort significantly, particularly during the initial phases, by identifying the viability of a particular converter design and reducing the number of simulation verification steps which are normally required.

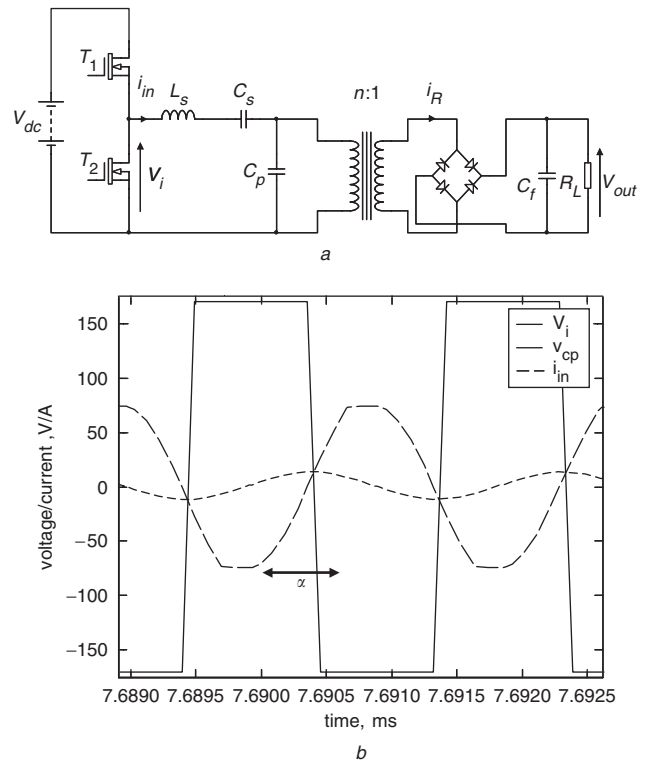
To reduce the complexity of the proposed design methodologies, the following assumptions are made:

- (i) All the passive components behave linearly;
- (ii) Diodes are ideal one-way switches with an on-state voltage drop;
- (iii) MOSFETs are ideal switches with zero on-state resistance;
- (iv) All switches commute instantaneously; and
- (v) The converter input voltage is constant.

## 2 Equivalent-circuit modelling of LCC voltage-output converter

Consider the LCC voltage-output converter shown in Fig. 1 in which the half-bridge MOSFETs  $T_1$  and  $T_2$  are switched in antiphase at a frequency  $f_s$ , thereby effectively ‘chopping’ the DC link voltage  $V_{dc}$  to produce a square-wave input voltage  $V_i$  to the resonant tank. This excites the resonant tank and causes current  $i_{in}$  to flow. The output voltage  $V_{out}$  is developed from the portion of  $i_{in}$  which flows into the filter capacitor  $C_f$  when the rectifier is conducting, during which time the parallel capacitor voltage  $v_{cp}$  is clamped to  $V_{out}$ . When  $i_{in}$  crosses zero, the rectifier turns off and  $v_{cp}$  falls (or rises) from  $V_{out}$  ( $-V_{out}$ ) with a negative (positive) slope. When  $v_{cp} = -V_{out}$ , the rectifier commutates and clamps  $v_{cp}$  at  $-V_{out}$ . The rate at which  $v_{cp}$  charges/discharges is determined by the values of  $C_p$  and  $R_L$ .

The behaviour of the LCC converter is similar to that of the classical series-resonant converter [1, 3], with the exception that the rate-of-change of voltage at the input



**Fig. 1** LCC voltage-output converter  
a Converter  
b Component waveforms

to the bridge rectifier is limited by the presence of  $C_p$ . Moreover, it can be shown [6–9] that the interaction between  $C_p$  and  $C_f$  via the rectifier can produce a voltage gain greater than unity, a feature which is not possible with a classical second-order series converter.

Although the series-resonant converter is readily analysed using FMA by replacing  $V_{dc}$  by an equivalent sinusoidal voltage and representing the load and rectifier by an equivalent resistor  $R_e$ , in practice this is not appropriate for the LCC voltage-output-converter variant since  $v_{cp}$  cannot be assumed to have either a square or sinusoidal waveform, owing to the  $dv/dt$ -limiting effect of  $C_p$ . Consequently, a describing function is used to model  $v_{cp}$  as a complex voltage source, that is ultimately transformed into an ‘equivalent’ complex impedance. This paper follows the approaches taken in [7, 8], with the exception that equivalent-circuit passive components are derived explicitly and the effect of the rectifier on-state voltage is considered. The proposed analysis begins by assuming that the tank-current waveform is sinusoidal, i.e.

$$i_{in} = \hat{I}_{in} \sin(\omega_s t) \quad (1)$$

where  $\hat{I}_{in}$  is the peak value of  $i_{Ls}$  and  $\omega_s = 2\pi f_s$  is the angular switching frequency. This is justified since the filtering action of the tank circuit removes high-order harmonics, effectively forcing the input current to be a sinusoid.

While the rectifier is conducting, the parallel capacitor voltage is clamped to the output voltage, and is described by

$$v_{cp} = n(V_{out} + kV_d) \text{sgn}(i_{Ls}) \quad (2)$$

where  $k = 1$  for a half-bridge rectifier or  $k = 2$  for a full-bridge rectifier.

From Fig. 1b, the rectifier switches off when the resonant-circuit current,  $i_{in}$ , crosses zero, at which time  $i_{in}$  starts to charge  $C_p$ . The period during which  $C_p$  is charging

can be obtained from the solution of

$$v_{cp} = v_{cp,0} + \frac{1}{C_p} \int_0^{\delta_1} \hat{I}_{in} \sin(\omega_s t) dt \quad (3)$$

where  $\delta_1$  defines the end of the diode nonconduction period and  $v_{cp,0}$  is the initial voltage on  $C_p$ . Solving for  $v_{cp}$  for the case when the initial condition is  $v_{cp,0} = -nV_b = -n(V_{out} + kV_d)$  gives

$$v_{cp} = -nV_b + \frac{\hat{I}_{in}}{\omega_s C_p} \{1 - \cos(\omega_s t)\} \quad (4)$$

The charging time (or nonconduction period) can be found by solving (4) for  $\delta_1$  to give

$$\delta_1 = \frac{1}{\omega_s} \cos^{-1} \left( 1 - \frac{2n\omega_s V_b C_p}{\hat{I}_{in}} \right) = \frac{1}{\omega_s} \theta_1 \quad (5)$$

where  $\theta_1 = \cos^{-1} \{1 - (2n\omega_s V_b C_p / \hat{I}_{in})\}$ .

The converter's output current  $I_{out}$  is obtained by determining the average current flowing through the rectifier during a complete cycle. Defining the angle at which the rectifier starts to conduct as  $\theta_1$ , the output current is found from

$$\begin{aligned} I_{out} &= \frac{n}{2\pi} \left\{ \int_{\theta_1}^{\pi} \hat{I}_{in} \sin(\theta) d\theta + \int_{\theta_1+\pi}^{2\pi} -\hat{I}_{in} \sin(\theta) d\theta \right\} \\ &= \frac{2n}{\pi} (\hat{I}_{in} - V_b \omega_s C_p) \end{aligned} \quad (6)$$

The output voltage is then readily determined from Ohm's law:

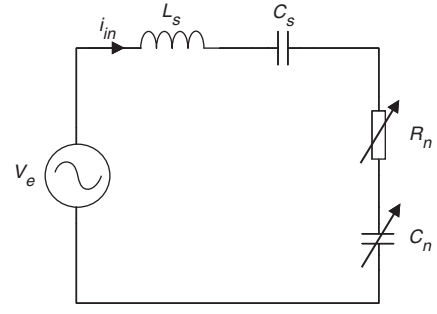
$$V_{out} = 2nR_L \left( \frac{\hat{I}_{in} - kn\omega_s C_p V_d}{\pi + 2n^2 R_L \omega_s C_p} \right) \quad (7)$$

To calculate  $V_{out}$ ,  $\hat{I}_{in}$  must be determined from the impedance that the converter presents to the source. To facilitate this,  $v_{cp}$  is converted into a complex voltage source using describing-function techniques. Using (2)–(5),  $v_{cp}$  is described over a full cycle by

$$v_{cp} = \begin{cases} -nV_b + \frac{\hat{I}_{in}}{\omega_s C_p} \{1 - \cos(\theta)\} & 0 \leq \theta < \theta_1 \\ nV_b & \theta_1 \leq \theta < \pi \\ nV_b - \frac{\hat{I}_{in}}{\omega_s C_p} \{1 + \cos(\theta)\} & \theta_1 \leq \theta < \theta_1 + \pi \\ -nV_b & \theta_1 + \pi \leq \theta < 2\pi \end{cases} \quad (8)$$

The describing function for  $v_{cp}$  is obtained by extracting the fundamental from the Fourier series of (8). Substituting for  $V_b$  and  $\theta_1$  and subsequently dividing by  $\hat{I}_{in}$  gives the equivalent impedance  $Z_e$  seen by the resonant circuit. Separating the equivalent impedance  $Z_e$  into a resistive component  $R_\eta$  and a capacitive component  $C_\eta$  gives

$$\begin{aligned} R_\eta &= \frac{4(2nR_L \hat{I}_{in} + k\pi V_d)(\hat{I}_{in} - 2n\omega_s C_p V_d)}{\gamma^2 \hat{I}_{in}^2} \\ C_\eta &= \pi \gamma C_p \hat{I}_{in} \left/ \left( \frac{2}{\gamma \hat{I}_{in}} \sqrt{\{n\pi\omega_s C_p(2nR_L \hat{I}_{in} + k\pi V_d)(\hat{I}_{in} - 2n\omega_s C_p kV_d)\}} \right. \right. \\ &\quad \times \{I_{in}(\gamma - 2\pi) + 2n\pi\omega_s C_p kV_d\} \\ &\quad \left. \left. + I_{in} \gamma \left[ \pi - \cos^{-1} \left\{ \frac{(\gamma - 2\pi)\hat{I}_{in} + 2n\pi\omega_s C_p kV_d}{\gamma \hat{I}_{in}} \right\} \right] \right) \right) \end{aligned} \quad (9)$$



**Fig. 2** Equivalent circuit for LCC voltage-output converter

where  $\gamma = \pi + 2\omega_s C_p n^2 R_L$ . Further information regarding the derivation of  $R_\eta$  and  $C_\eta$  is given in [8].

Inserting the series combination of  $R_\eta$  and  $C_\eta$  in place of  $C_p$  and its parallel components yields the equivalent series circuit for the converter shown in Fig. 2.

From standard circuit analysis, the total impedance  $Z_{tot}$  is given by

$$\begin{aligned} Z_{tot} &= Z_e + j \left( \omega_s L_s - \frac{1}{\omega_s C_s} \right) \\ &= R_\eta + j \left( \omega_s L_s - \frac{C_s + C_\eta}{\omega_s C_s C_\eta} \right) \\ &= R_\eta + j \left( \omega_s L_s - \frac{1}{\omega_s C_{tot}} \right) \end{aligned} \quad (10)$$

Hence, the magnitude of the input current,  $\hat{I}_{in}$ , is obtained from Ohm's law as

$$\hat{I}_{in} = \frac{V_e}{|Z_{tot}|} \quad (11)$$

The output voltage is obtained by substituting (11) into (7).

For analysis purposes, the calculation of  $C_\eta$  and  $R_\eta$  is reliant on  $\hat{I}_{in}$ , which is not generally known *a priori*. However, by setting  $V_d = 0$  in (5), (9), (10) and (11), an initial estimate for  $\hat{I}_{in}$  (denoted with a prime) is obtained iteratively, as follows:

$$\begin{aligned} R'_\eta &= \frac{8n^2 R_L}{\gamma^2} \\ C'_\eta &= \frac{\pi \gamma^2 C_p}{2n \sqrt{(2\pi\omega_s C_p R_L)(\gamma - 2\pi) + \gamma^2 \left\{ \pi - \cos^{-1} \left( \frac{\gamma - 2\pi}{\gamma} \right) \right\}}} \end{aligned} \quad (12)$$

The total impedance seen at the source is therefore estimated to be

$$Z_{tot} = R'_\eta + j \left( \omega_s L_s - \frac{1}{\omega_s C'_{tot}} \right) \quad (13)$$

An initial estimate of the input current immediately follows from (13), i.e.

$$\hat{I}'_{in} = \frac{(2/\pi) V_{dc}}{\sqrt{\left\{ R'^2_\eta + \left( \omega_s L_s - \frac{1}{\omega_s C'_{tot}} \right)^2 \right\}}} \quad (14)$$

Once  $\hat{I}'_{in}$  has been determined, an estimate of  $V_{out}$  can subsequently be refined using the following

procedure:

- Substitute  $\hat{I}'_{in}$  and  $V_d$  into (5), (9), (10) and (11) to obtain a revised value for  $\hat{I}_{in}$ .
- Return to step 1 for the next iteration, or after convergence go to step 3.
- Finally, use the refined value of  $\hat{I}_{in}$  to determine  $V_{out}$  using (7).

Numerous simulation studies have shown that the foregoing iterative refinement procedure converges rapidly, with very little improvement in the results after the third iteration. A more detailed description of the equivalent-circuit model is given in [9].

### 3 Design methodologies

This Section describes five novel design methodologies based on the equivalent-circuit model presented. In addition, a procedure for incorporating the magnetising inductance of a transformer within the design of a converter, along with a procedure for filter capacitor sizing, are also discussed.

The first methodology (designated DM1) bases the design on standard AC-circuit parameters ( $Q$  and  $\omega_0$ ) and the rectifier nonconduction angle  $\theta_1$ . The second methodology (DM2) employs the definition of the switch power factor, whilst the remaining methods (DM3–5) allow some resonant-circuit values to be specified during the design process, thereby allowing the designer to use standard ‘off-the-shelf’ components.

As with most design techniques, a standard set of core-specification parameters is required, summarised in Table 1.

**Table 1: Standard specification parameters**

Parameter	Description
$V_{dc}$	DC input voltage (V)
$V_{out}$	DC output voltage (V)
$P_{out}$	Output power (W)
$V_d$	Diode on-state voltage (V)

#### 3.1 Design method 1 (DM1): specified rectifier nonconduction angle $\theta_1$

In this method, all the resonant-circuit components (i.e.  $L_s$ ,  $C_s$  and  $C_p$ ) of the converter are determined for a specific rectifier nonconduction angle  $\theta_1$ , a given operating frequency  $f_s$  and a combined equivalent resonant frequency  $f_0$ . Unlike the previously published techniques presented in [6, 7], the resonant-circuit-capacitor ratio ( $C_p/C_s$ ) is not required, thereby making the interpretation of the design more tangible. However, before presenting the design algorithm in detail, it is instructive to consider how the behaviour of the converter is influenced by these parameters.

*Analysis:* In a similar manner to the normalising procedure described in [3], the parameters  $Q$ ,  $\omega_0$  and  $\omega_n$  can be used to derive an expression for the voltage-conversion ratio  $M_v = V_o/V_{dc}$ , where

$$Q = \frac{1}{\omega_0 C_{tot} R_\eta} = \frac{\omega_0 L_s}{R_\eta}, \quad \omega_0 = \frac{1}{\sqrt{(L_s C_{tot})}}, \quad \omega_n = \frac{\omega_s}{\omega_0} \quad (15)$$

Specifically, from [3],  $M_v$  is obtained by combining (14) and (7)—neglecting  $V_{ds}$ , substituting into (15) and rearranging using (12), to give:

$$M_v = \frac{V_{out}}{V_{dc}} = \frac{1}{n\{1 + \cos(\theta_1)\}} \frac{1}{\sqrt{\left\{1 + Q^2 \left(\omega_n - \frac{1}{\omega_n}\right)\right\}^2}} \quad (16)$$

By inspection, it can be seen that (16) has a similar form to the standard series-resonant-converter FMA model [1, 3], with the exception that the gain is modified by the term  $1/[n\{1 + \cos(\theta_1)\}]$ , which, importantly, shows the potential for having a voltage gain greater than unity, as will be evident from Fig. 3. Moreover, from Fig. 3, it can be seen that the voltage gain increases with the rectifier nonconduction angle. This is expected since, when the rectifier is not conducting,  $C_p$  is being charged and can therefore support a greater voltage if charged for longer. Furthermore, Fig. 3 shows that, while the magnitude of the voltage gain at resonance is unaffected by the  $Q$ -factor, the bandwidth, and hence the rate at which the voltage gain changes with frequency, is significantly affected. This imposes restrictions on the maximum value of  $Q$  that can be employed in terms of the converter controllability and sensitivity to component tolerances.

Although Fig. 3 provides useful information regarding the voltage transfer and control characteristics of an LCC converter, the designer is invariably also required to optimise converter efficiency. However, rather than entering into a detailed discussion about the resonant tank, transformer and rectifier efficiencies (details of which can be found elsewhere [1]), the ratio of the resonant-circuit current to the output current  $M_i$  will be used as a metric for design. Setting  $V_d = 0$  in (7), and substituting  $V_{out} = I_{out} R_L$ , using RMS quantities and rearranging, gives

$$M_i = \frac{\hat{I}_{in,rms}}{I_{out}} = \frac{\pi}{\sqrt{2n\{1 + \cos(\theta_1)\}}} \quad (17)$$

Again, by ignoring the transformer turns ratio, it can be seen that  $M_i$  is dependent only on  $\theta_1$ . From Fig. 4, it can clearly be seen that the resonant-circuit current increases significantly for  $\theta_1 > 90^\circ$ . Further, from Fig. 3, it can be seen that, to achieve a voltage gain greater than unity at resonance,  $\theta_1 > 90^\circ$ .

To summarise,  $Q$  should be chosen to reduce problems associated with component-tolerance sensitivity and controllability issues, and  $\theta_1$  should be chosen to reduce the circulating current in the tank circuit, thereby reducing conduction losses in the switches and inductor, and to minimise voltage/current stresses on individual components. Consequently, for a step-down converter,  $\theta_1 < 90^\circ$ , whilst for step-up operation, the designer must trade off the effects of an increase in transformer turns ratio with circulating-current levels. A suitable value for  $\theta_1$ , having been chosen, together with nominal values for  $\omega_s$  and  $\omega_0$ , specific component values can be determined.

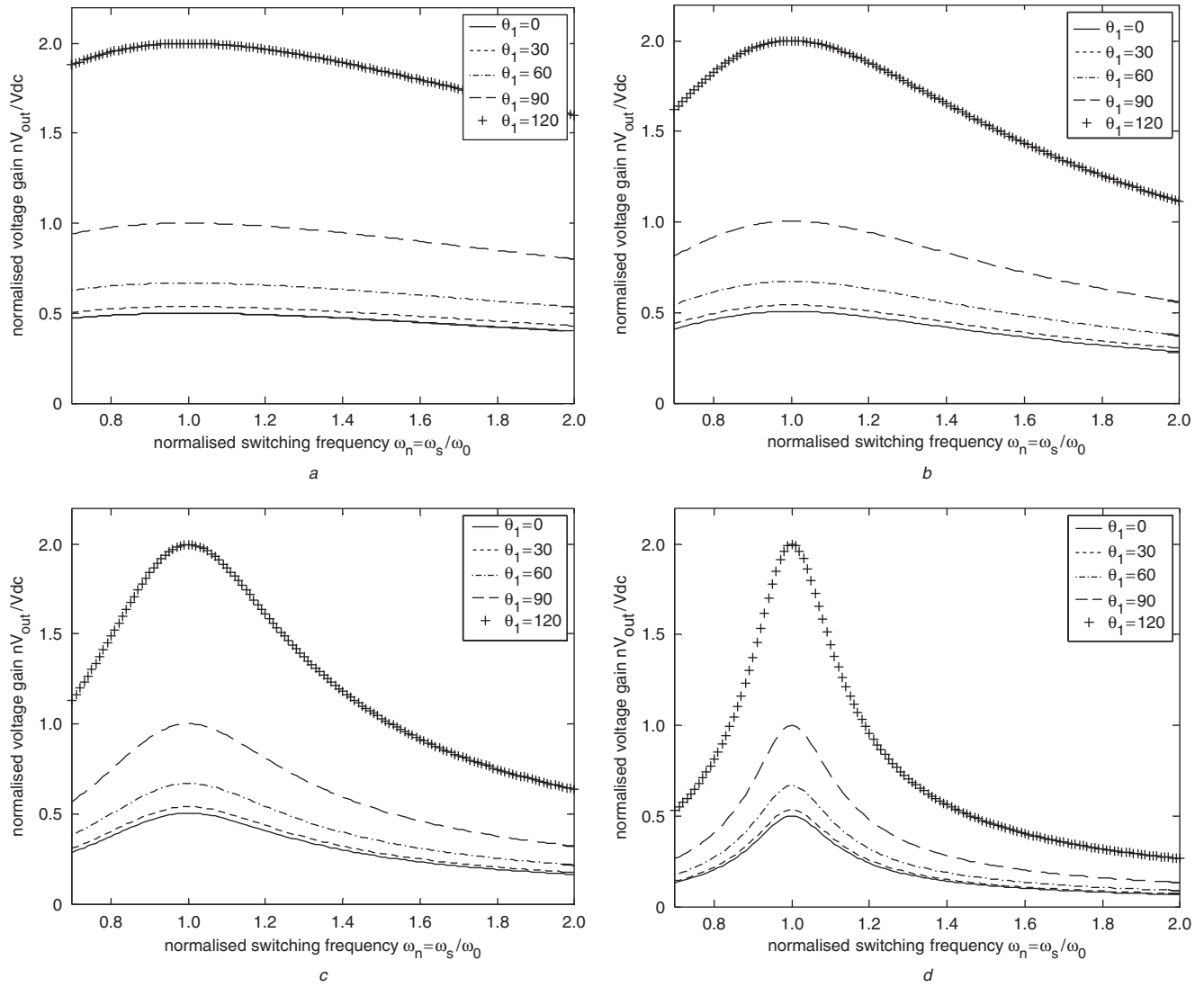
*Design procedure:* The load resistance is calculated from

$$R_L = \frac{V_{out}}{I_{out}} \quad (18)$$

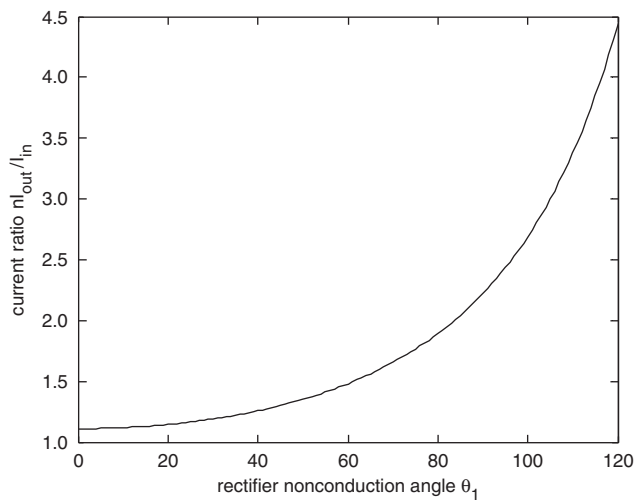
where  $I_{out} = P_{out}/V_{out}$ . A value for  $C_p$  is then obtained by solving (6) for  $\hat{I}_{in}$ , and substituting into (5). After rearrangement, this gives

$$C_p = \frac{\pi I_{out}}{2n\omega_s V_b} \frac{1 - \cos(\theta_1)}{2n - 1 + \cos(\theta_1)} \quad (19)$$





**Fig. 3** Converter-voltage transfer ratio for varying  $Q$ -factors  
*a*  $Q=0.5$   
*b*  $Q=1$   
*c*  $Q=2$   
*d*  $Q=5$



**Fig. 4** Resonant-circuit current-to-output current ratio

Since the equivalent-circuit components  $R_\eta$  and  $C_\eta$  are defined entirely in terms of the design parameters  $f_s$ ,  $R_L$  and  $C_p$ , they can immediately be found from (9).

Subsequently,  $L_s$  and  $C_s$  are determined from the required impedance of the equivalent resonant circuit which is necessary to achieve a specific input current, where

$$\hat{I}_{in} = \frac{\pi I_{out}}{n\{1 + \cos(\theta_1)\}} \quad (20)$$

The total circuit capacitance,  $C_{tot}$ , is obtained by rearranging (14) and substituting into (15) to give

$$C_{tot} = \frac{\omega_n^2 - 1}{\omega_s \sqrt{\left\{ \frac{4}{\pi^2} \left( \frac{V_{dc}}{\hat{I}_{in}} \right)^2 - R_\eta^2 \right\}}} \quad (21)$$

Since  $C_\eta$  is obtained from (9),  $C_s$  can be determined from

$$C_s = \frac{C_\eta C_{tot}}{C_\eta - C_{tot}} \quad (22)$$

Finally, the series inductance is found from the definition of the converter resonant frequency (15), namely

$$L_s = \frac{1}{\omega_0^2 C_{tot}} \quad (23)$$

### 3.2 Design method 2 (DM2): specified switch power factor

In this approach, the resonant-circuit component values are calculated to achieve a specified switch power factor at the specified switching frequency, given a maximum voltage stress on  $C_s$ .

An initial estimate for the peak resonant current  $\hat{I}'_{in}$  is obtained from the output power, the power factor  $P_f = \cos \phi$  and the RMS value of the input voltage is  $(\sqrt{2}/\pi)/V_{dc}$ , as follows:

$$\hat{I}'_{in} = \frac{\pi P_{out}}{V_{dc} P_f} \quad (24)$$

The power loss  $P_{loss}$  in the circuit is then estimated from

$$P_{loss} = 2V_d I_{out} + \frac{\hat{I}'_{in}{}^2 R_s}{2} \quad (25)$$

where  $R_s$  is an 'optional' series resistance. The total input power is then obtained from

$$P_{in} = P_{out} + P_{loss} \quad (26)$$

By replacing  $P_{out}$  with  $P_{in}$  in (24), a revised value for  $\hat{I}'_{in}$  is obtained as

$$\hat{I}_{in} = \frac{\pi P_{in}}{V_{dc} P_f} \quad (27)$$

The improved estimate of  $\hat{I}_{in}$  is then substituted into (25), and the refinement process is repeated. Again, it has been shown that a maximum of three iterations is usually sufficient to obtain results which are accurate to three significant figures.

$R_\eta$  and  $C_\eta$  are found from (9), and  $C_p$  is obtained by rearranging (6):

$$C_p = \frac{\hat{I}_{in}}{\omega_s V_b} - \frac{\pi I_{out}}{2n\omega_s V_b} \quad (28)$$

$C_s$  is obtained by specifying the maximum allowable voltage stress  $\hat{V}_{cs}$  and the peak tank current:

$$C_s = \frac{\hat{I}_{in}}{\omega_s \hat{V}_{cs}} \quad (29)$$

giving the total equivalent-circuit capacitance:

$$C_{tot} = \frac{C_s C_\eta}{C_s + C_\eta} \quad (30)$$

$L_s$  is then found from (23).

### 3.3 Design method 3 (DM3): specified $C_p$

The remaining design methodologies are particularly useful if the designer is constrained in the choice of component values. In this particular case,  $L_s$  and  $C_s$  are calculated when the value of  $C_p$  is fixed. The method is based on DM1 in which the peak input current was used to determine  $L_s$  and  $C_s$ .

Initially,  $R_L$  is calculated from (18) and the peak input current  $\hat{I}_{in}$  is found by rearranging (6) and inserting values for the switching frequency and  $C_p$ :

$$\hat{I}_{in} = \frac{\pi I_{out} + 2nV_b \omega_s C_p}{2n} \quad (31)$$

Values of  $L_s$  and  $C_s$  are then determined from (21)–(23).

### 3.4 Design method 4 (DM4): specified $L_s$

To begin with, a value of  $C_p$  is selected based on either DM1 or DM2;  $C_s$  is then readily determined from  $L_s$ ,  $\omega_s$  and  $\omega_0$ :

*Method 1: Specified diode non-conduction angle*

Using  $\theta_1$ ,  $C_p$  is obtained directly from (19).

*Method 2: Specified switch power factor*

The peak input current is found by solving (25)–(27) iteratively. Once  $\hat{I}_{in}$  has converged,  $C_p$  is calculated using (31).

$C_p$  having been determined the total circuit capacitance is given by:

$$C_{tot} = \frac{1}{\omega_0^2 L_s} \quad (32)$$

from which  $C_s$  is found by substituting (9) into (22).

### 3.5 Design method 5 (DM5): specified $C_s$ and $C_p$

In this method, a value of  $L_s$  is sought from the specification of  $C_p$ ,  $C_s$ ,  $\omega_s$  and  $\omega_0$  by employing (25)–(27) to determine the input current in an iterative manner. Subsequently, (9) is used to calculate the equivalent components  $R_\eta$  and  $C_\eta$ . The total circuit capacitance is then deduced from (30) and, finally, the required value of  $L_s$  is determined from (32).

Although a designer may elect to use any of the foregoing design methodologies to determine the tank components for a specific resonant converter, certain methodologies lend themselves to particular applications. Specifically, methodology DM1 is appropriate when attempting to minimise the EMI generated by a converter, since the rectifier nonconduction angle  $\theta_1$  determines the noise level which is generated by the rectifier. For small values of  $\theta_1$ , the waveform of  $v_{cp}$  resembles a square wave, and therefore has a large harmonic content. However, for large values of  $\theta_1$ ,  $v_{cp}$  tends to a sinusoid with a much lower harmonic content. Thus, a designer can vary  $\theta_1$  so as to minimise EMI. Methodology DM2 is more appropriate when efficiency is a key consideration, since the procedure explicitly accounts for the power losses associated with the rectifier diodes, the MOSFET on-state resistance and other parasitic resistances which are present in the tank circuit at the design stage. This methodology also allows the maximum voltage across  $C_s$  to be specified during the design process. Design methodologies DM3–DM5 would be employed when a designer is required to use 'off-the-shelf' circuit components.

### 3.6 Inclusion of transformer parasitics

Although an LCC converter can be designed to provide a voltage-boost capability, the requirement for a significant step-up voltage conversion will often require the incorporation of a transformer and, from a commercial perspective, the use of an isolation transformer is often required for reasons of safety. The effects of transformer magnetising inductance  $L_m$  can be included in a design by modifying tank-circuit-current calculation. This is accomplished by:

- (i) Initially, neglect  $L_m$  and calculate an initial value,  $\hat{I}_{in\_initial}$ , of  $\hat{I}_{in}$  from (20), (24) or (31), depending on the design methodology.
- (ii) Using  $\hat{I}_{in}$ , determine values for the equivalent components  $R_\eta$  and  $C_\eta$  using (9).
- (iii) Calculate the fundamental component of  $v_{cp}$ , from  $v_{cp-1} = \hat{I}_{in} (R_\eta + \frac{1}{sC_\eta})$ , where  $s \rightarrow j\omega_s$ .
- (iv) Determine the current drawn by  $L_m$ , from  $I_{Lm} = \frac{v_{cp-1}}{j\omega_s L_m}$ .
- (v) Calculate a revised value  $\hat{I}_{in\_new}$  for the input current, from  $\hat{I}_{in\_new} = \hat{I}_{in\_initial} + \hat{I}_{Lm}$ .
- (vi) Determine the resonant-circuit components, as before, using  $\hat{I}_{in\_new}$ .

### 3.7 Filter-capacitor sizing

In general, the value of  $C_f$  is chosen to satisfy the output-voltage-ripple specification, such that the ripple does not exceed  $\alpha\%$ . A suitable value for  $C_f$  is calculated by considering the charge which flows into  $C_f$  and the corresponding voltage rise over a specific time interval. From the example waveforms shown in Fig. 5, it can be seen that, during the rectifier conduction period, the current which flows into  $C_f$  is the difference between the rectifier input and output currents.

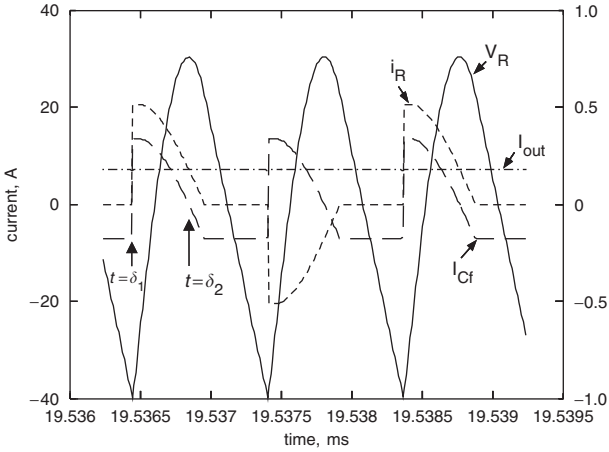


Fig. 5 Voltage and current waveforms for filter capacitor

Considering the period during which current is replenishing the charge on  $C_f$ , then

$$Q_{Cf} = \int_{\delta_1}^{\delta_2} i_{Cf} dt = \int_{\delta_1}^{\delta_2} \{n\hat{I}_{in} \sin(\omega_s t) - I_{out}\} dt \quad (33)$$

where  $\delta_2$  is the time at which  $i_{Cf}$  becomes zero.  $\delta_2$  is found from the solution of:

$$i_{Cf} = n\hat{I}_{in} \sin(\omega_s t) - I_{out} = 0$$

Therefore

$$\delta_2 = \frac{1}{\omega_s} \left\{ \pi - \sin^{-1} \left( \frac{I_{out}}{n\hat{I}_{in}} \right) \right\} \quad (34)$$

During this period  $V_{out}$  increases by an amount  $v_{rip}$ , which provides a further expression for the charge flowing into  $C_f$ :

$$Q_{Cf} = C_f v_{rip} = C_f \alpha V_{out} \quad (35)$$

The minimum value of  $C_f$  which is required to limit  $v_{rip}$  to a specified level  $\alpha$  is found by equating (33) and (35), and rearranging to give

$$C_f > \frac{n\hat{I}_{in}}{\alpha V_{out} \omega_s} \{ \cos(\omega_s \delta_1) - \cos(\omega_s \delta_2) \} - \frac{1}{\alpha R_L} (\delta_2 - \delta_1) \quad (36)$$

If  $V_{out} \gg V_{ds}$  (36) reduces to

$$C_f > \frac{1}{2\alpha R_L \omega_s} \left[ \sqrt{(\gamma^2 - 4)} - \gamma + 2 \left\{ \cos^{-1} \left( \frac{\gamma - 4R_L \omega_s C_P}{\gamma} \right) + \sin^{-1} \left( \frac{2}{\gamma} \right) \right\} \right] \quad (37)$$

### 3.8 Experimental verification

To demonstrate the validity of the proposed design methodologies, a 25 V-input, 35 V-output, 22 W experimental converter was designed to meet the specification outlined in Table 2 using DM4. The component values for the remaining resonant-circuit components were determined, using DM4, as  $C_p = 89$  nF,  $C_s = 224$  nF while the value for  $C_p$  was realised by paralleling four 22 nF capacitors to give 87 nF measured and the value for  $C_s$  created from a 220 nF capacitor was connected in parallel with two 2.2 nF capacitors giving 224 nF measured.

Table 2: Prototype converter parameters

Parameter	$V_{dc}$ (V)	$V_{out}$ (V)	$P_{out}$ (W)	$L_s$ ( $\mu$ H)	$f_s$ (kHz)	$f_{0s}$ (kHz)	$Q$	$\theta_1$ (deg)	$n$
Value	25	35	22	18.4	150	136	5.5	120	1

Operating as specified (i.e. with  $f_s = 150$  kHz), the converter had an output voltage of 33.6 V which was within 5% of the specification. Figure 6 shows the time-domain waveforms of the input voltage to the tank ( $V_i$ ) and the parallel capacitor voltage ( $v_{Cp}$ ). Inspection of Fig. 6 shows the diode nonconduction period  $\theta_1$  to be  $\sim 123^\circ$ , which is close to the desired  $120^\circ$ .

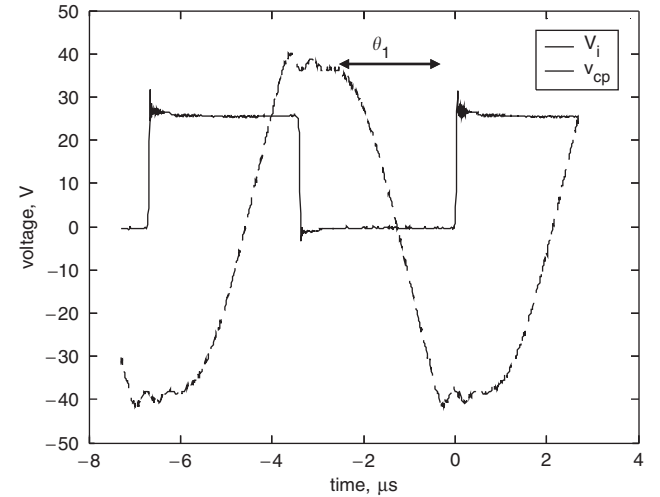


Fig. 6 Voltage waveforms for prototype 22 W LCC voltage-output converter

## 4 Accuracy bounds

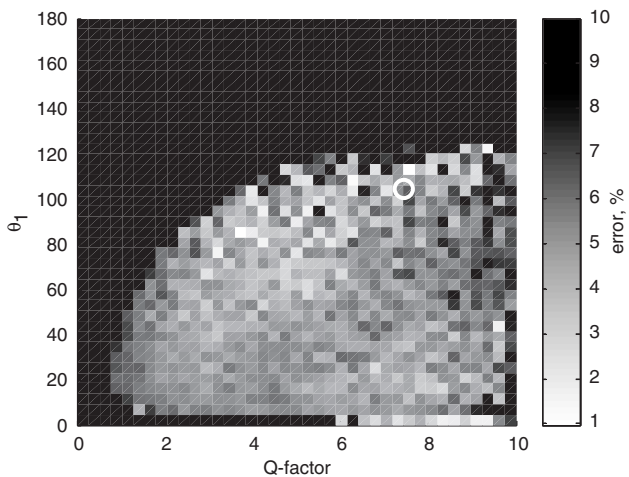
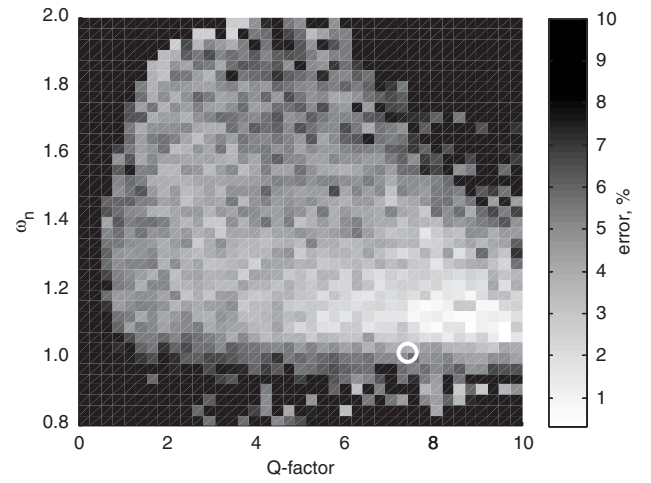
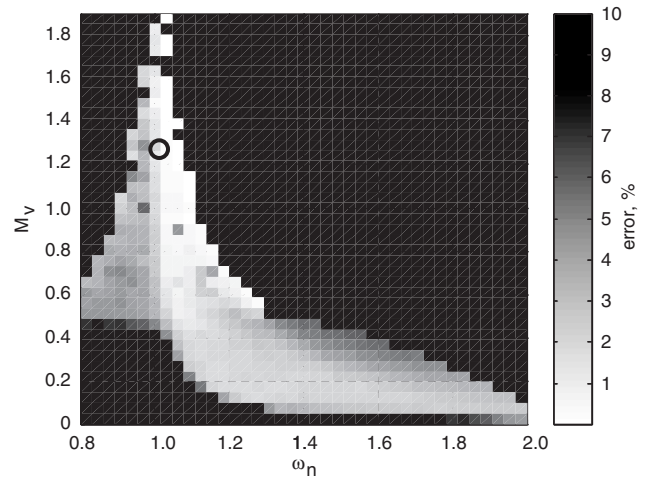
Classically, all FMA-based design methodologies assume that voltage and current waveforms can readily be approximated to sinusoids or square waves to simplify circuit analysis. For the analysis described in this paper, the resonant circuit is assumed to respond only to the fundamental component of the input-voltage waveform (because of the filtering attributes of the tank) and a sinusoidal current is therefore assumed to flow in the resonant circuit. However, for switching frequencies much higher than the resonant frequency (and for particular load ranges), the tank current becomes distorted, tending to a triangular waveshape, and thereby contravenes the FMA sinusoidal assumption. Consequently, there exists only a subset of converter parameters, obtained from FMA results, that provide realisable converters having the expected performance characteristics (see the account given [1], for instance).

**Table 3: Confidence-map-parameter ranges**

Parameter	$V_{dc}$ (V)	$Q_s$	$C_p/C_s$	$Z_0$	$f_{0s}$ (kHz)	$f_s$	$V_d$ (V)
Value	1–100	1–10	0.01–100	0.01–100	100	$(1-2) \times f_r$	0.45

Here then, a comprehensive investigation is presented to determine parameters that affect prediction performance significantly, by comparing results from over 250 000 simulation studies, obtained from an accurate nonlinear state-variable model of the converter [10], with those from the equivalent-circuit model presented in Section 2, over a bounded range of typical parameter values—specifically,  $V_{dc}$ ,  $Z_0$ , the  $Q$ -factor, the ratio  $C_p/C_s$  and  $\omega_0$ . The ranges are summarised in Table 3. From the comparison of results, errors maps are generated to provide an indication of the expected accuracy of a specific converter design. Justification for using the nonlinear-state-variable model as a benchmark metric is that no restrictions regarding the circuit-voltage/current waveshapes or the interaction between the output filter, rectifier and tank, are present, and it is therefore not constrained by underlying assumptions of FMA.

By appropriate manipulation of the resulting data, so-called ‘2-D confidence maps’ that show the impact of each design parameter on the ‘confidence’ of a particular converter design giving the expected performance, are generated. Since the parameters are applied to the equivalent-circuit model presented in Section 2, the resulting confidence maps can be employed generically. In this case, the data are presented in a manner that can be applied iteratively in conjunction with design procedure DM1; see Figs. 7–9. Although it is recognised that this type of statistical analysis only indicates relative trends, the body of evidence suggests that clearly defined regions of parameter ranges exist where the designer can have a high ‘confidence’ in realising expected performance from a particular design procedure. For use with DM1, in this example, the diode voltage drop is assumed constant, (0.45 V) in each case. Note that the normalised parameters that have been selected in this case are among many that could have been chosen, including those relating to the ratios  $C_p/C_s$  and  $R_L/Z_0$ , among others.

**Fig. 7** Output voltage prediction error as a function of  $\theta_1$  and  $Q$ -factor**Fig. 8** Output voltage prediction error as a function of  $\omega_n$  and  $Q$ -factor**Fig. 9** Output voltage prediction error as a function of  $M_v$  and  $\omega_n$ 

The procedure for generating the confidence maps is summarised below:

- (i) Since,  $Q_s = \{(\pi^2/8R_L)\sqrt{(L_s/C_s)}\} = Z_0/R_L$ ,  $R_L$  is calculated.
- (ii) Since  $\omega_{os} = 1/\sqrt{(L_s/C_s)}$ ,  $C_s$  is calculated from,  $C_s = 1/\omega_{os}^2 Z_0$ .
- (iii) Using  $A = C_p/C_s$ ,  $C_p$  is calculated from  $C_p = AC_s$ .
- (iv) Finally,  $L_s$  is determined from  $L_s = Z_0^2 C_s$ .

The output-filter capacitor for each converter design is selected to limit the ripple on the output voltage to 1%. Finally, the nonlinear-converter model is simulated in the time domain for  $10 \times C_p R_L$  seconds to allow sufficient time to achieve steady-state operation.



The variable space on a confidence map is divided into a grid with each 'pixel' representing the level of prediction error (from 0 to 10%), the pixel value being determined by the average 'error' within each element of the grid. The prediction error is calculated from

$$V_{error} = \left| \frac{V_{out} - V_{out\_ss}}{V_{out\_ss}} \right| \times 100\% \quad (38)$$

where  $V_{out}$  and  $V_{out\_ss}$  are the output voltages obtained from the equivalent-circuit model and the state-variable model, respectively.

Figure 7 shows the percentage error between the reference state-variable model and the equivalent-circuit model as a function of the rectifier nonconduction angle  $\theta_1$  and the equivalent circuit  $Q$ -factor. It can be seen that there exists a definite area, denoted by the dark regions, in which the equivalent-circuit-model predictions become inaccurate. With reference to Fig. 7, to ensure that accurate design predictions can be made using the FMA-based design methodologies, values of  $Q > 4$  and  $\theta_1 < 120^\circ$  should be chosen. However, limiting the rectifier nonconduction angle  $\theta_1 < 120^\circ$  also restricts the maximum voltage gain of the converter to be less than 2 (see Fig. 3).

Figure 8 shows the prediction error as a function of the ratio of the switching frequency and the 'real' resonant frequency, denoted by  $\omega_n$ , and the equivalent circuit  $Q$ -factor. To have high confidence in realising a converter with the required attributes, it is evident that it should be designed to exhibit a relatively high  $Q$ -factor and, correspondingly, should be switched close to the resonant frequency. This conforms to intuitive expectations, since high  $Q$ -factor tank circuits provide low-distortion sinusoidal tank currents near resonance. However, as discussed previously, the designer must trade the effects of employing a high  $Q$ -factor against the effects on controllability and component sensitivity. Figure 9 shows the design confidence as a function of the voltage-conversion ratio  $M_v$  and the  $Q$ -factor, and reinforces these findings, since, for instance, confidence is highest for converter designs having  $M_v > 1$  and  $1 < \omega_n < 1.1$ , with the performance of most being within 1% of that predicted from state-variable simulations, as a consequence of the improved filtering of the excitation-voltage harmonics. Moreover, for converter designs requiring  $0.5 < M_v < 1$ , the results suggest that  $\omega_n$  should be  $< 1.3$ , and, for gains less than 0.5, higher values of  $\omega_n$  should be chosen to ensure reasonable confidence in realising the expected converter performance.

Although only a small sample of the possible error/confidence mappings have been presented (other key variants include confidence maps as functions of  $M_v$  against  $\theta_1$  and  $\theta_1$  against  $\omega_n$ ), it is apparent that they enable parameter boundaries to be viewed readily, thereby restricting the design space. Moreover, although confidence maps require significant computational effort to generate, ultimately, they are generic, and can be used as an integral part of an interactive design tool. In practice, this is achieved by simultaneously placing a marker on each map at intercepts which correspond to the converter design parameters. The underlying value on the confidence maps then indicates the prediction accuracy. A designer would then interactively modify the converter design parameters so as to achieve a design that satisfied the specification to

the required degree of confidence. By way of example, a converter design whose parameters are  $L_s = 47 \mu\text{H}$ ,  $C_s = 22 \text{ nF}$ ,  $C_p = 33 \text{ nF}$ ,  $f_s = 190 \text{ kHz}$ ,  $V_{dc} = 48 \text{ V}$ ,  $R_L = 70 \Omega$  and  $V_d = 0.45 \text{ V}$  is superimposed on the confidence maps in Figs. 7–9, from which it can be inferred that this particular design would have an estimated prediction error of approximately 5%. This figure is consistent with the predicted output voltage from the equivalent-circuit and state-variable models, being  $V_{out} = 63.3 \text{ V}$  and  $V_{out\_ss} = 65.9 \text{ V}$ , respectively, which correspond to an error of 4%.

## 5 Conclusions

The paper has presented new approaches for designing LCC voltage-output resonant converters. Specifically, five methodologies for determining the values of the resonant-circuit components, employing various key performance and circuit parameters, including the diode nonconduction angle and the switch power factor, have been investigated. Practical results from a prototype converter have demonstrated the accuracy of the proposed design methodologies. In addition, techniques to account for the effect of the transformer magnetising inductance on the circuit design, and for sizing the filter capacitor, based on an equivalent-circuit model, have been proposed.

The paper has also applied the concept of error mapping. By simulating over 250 000 LCC converter designs over a constrained parameter space, maps of prediction accuracy against design variables have been presented, and their use as an integral part of an interactive design tool has been discussed. Note that, although only a limited number of confidence maps have been included, as a demonstration of principle, ultimately, their generation and use is generic to a wider parameter space, and they can be derived for all other resonant converter variants.

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